Optimization for Quantum Systems

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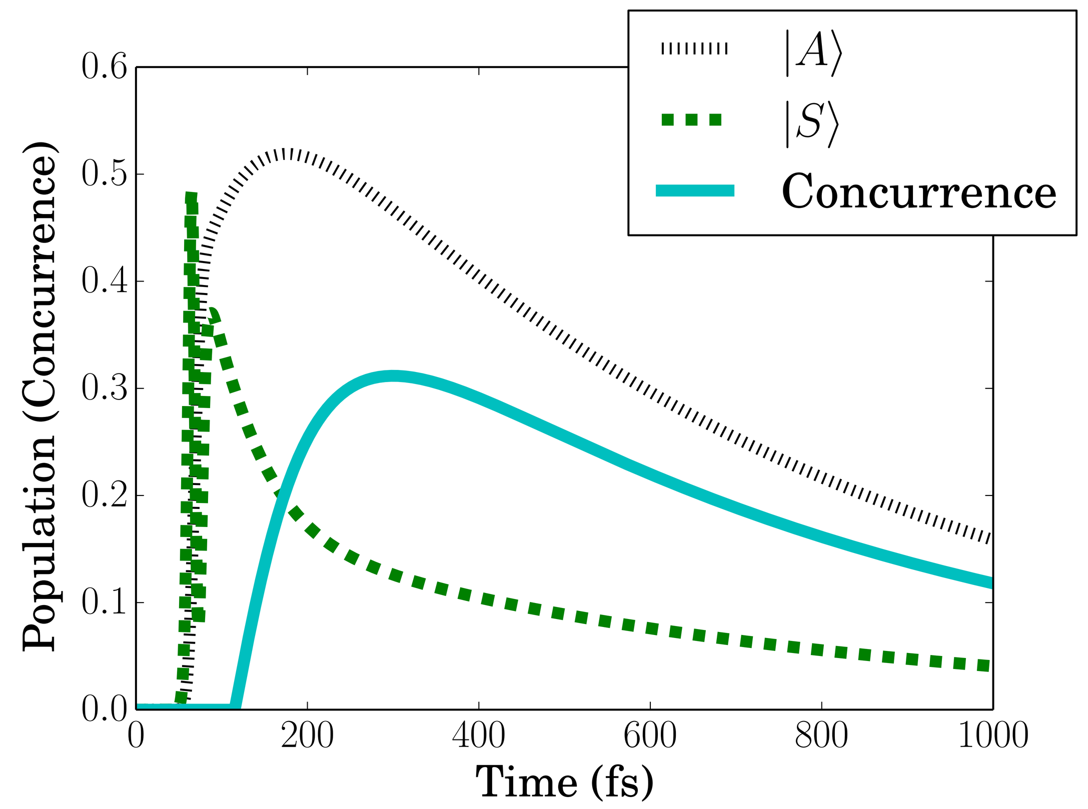
Maximizing concurrence

- Concurrence is a measure of entanglement of a quantum system
- Concurrence of two quantum dots excited by a single optical laser pulse:



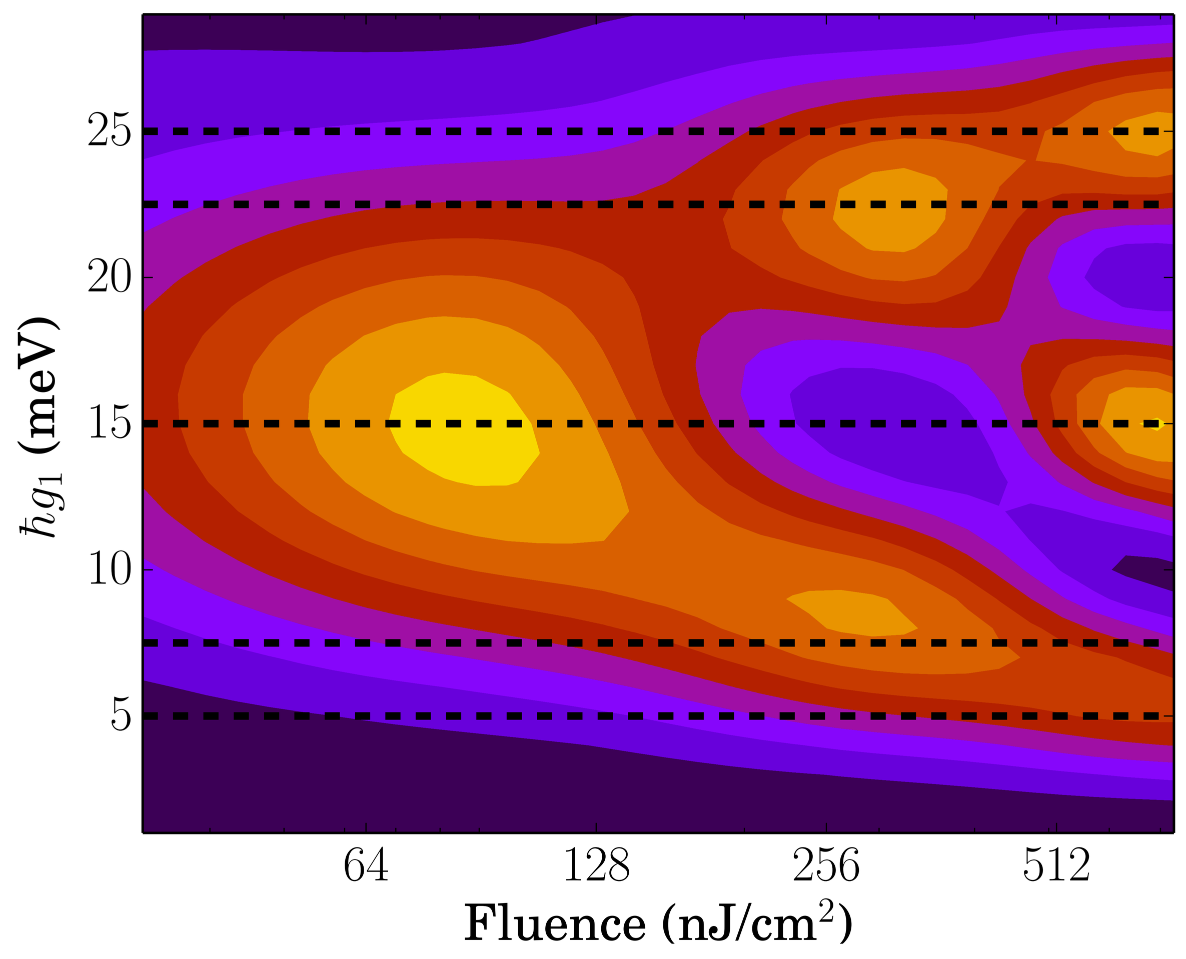
- Pairwise concurrence is measured by

$$C_{ij} = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\},$$

where λ_k are the eigenvalues of a density matrix relating particles i and j

- Want to identify system parameters x solving

$$\underset{x}{\text{maximize}} \sum_{ij} C_{ij}(x)^2$$



QAOA

Quantum approximate optimization algorithm

- Begins with an initial state $|+\rangle^{\otimes n}$
- Quantum evolution is performed by applying two alternating operators based on the cost Hamiltonian H_C and mixing Hamiltonian H_M

$$|\psi(\theta)\rangle = |\psi(\beta,\gamma)\rangle = e^{-i\beta_p H_M} e^{-i\gamma_p H_C} \cdots e^{-i\beta_1 H_M} e^{-i\gamma_1 H_C} |+\rangle^{\otimes n}$$

- The objective function is the energy of H_C in the state $|\psi(\beta,\gamma)\rangle$:

$$f(\beta,\gamma) = -\langle\psi(\beta,\gamma)| H_C |\psi(\beta,\gamma)\rangle$$

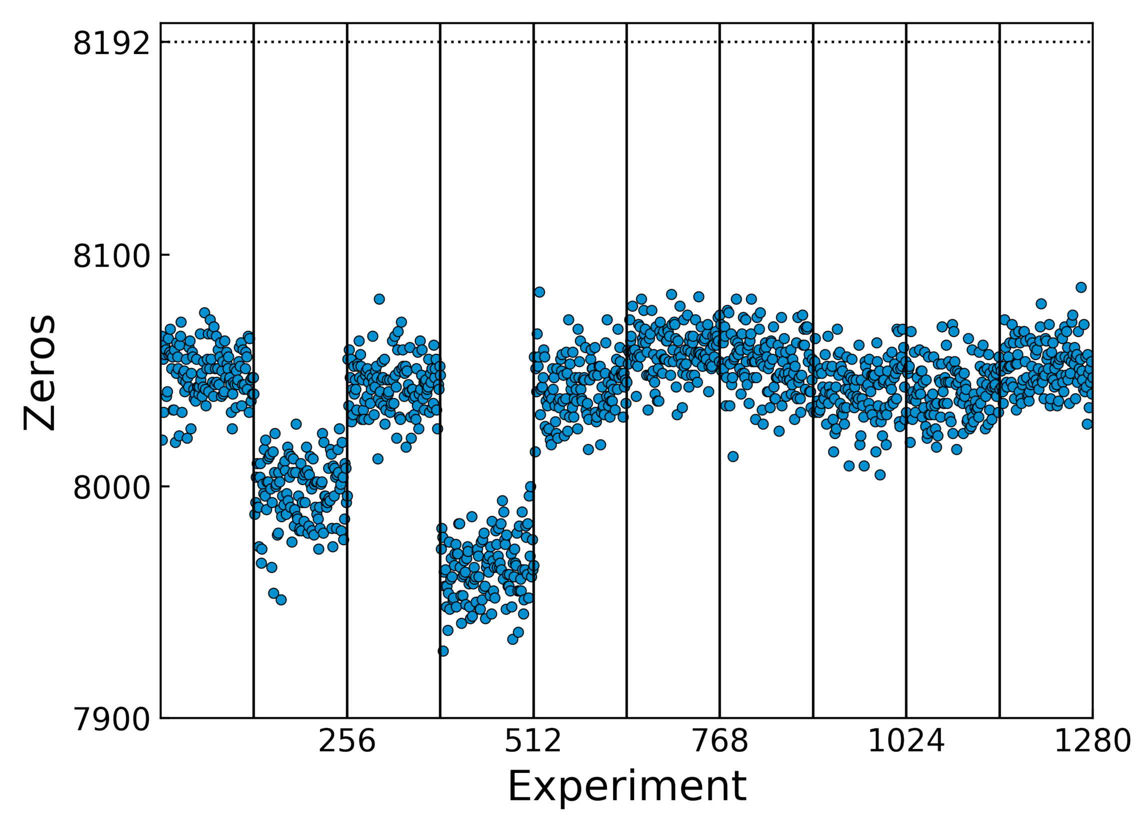
- For a small number of steps, p, the quantum evolution can be performed on NISQ devices
- QAOA's performance depends critically on the quality of angles of rotation produced by the classical optimizer
- The QAOA objective contains many suboptimal local optima. This motivates the use of a multistart framework

Time-varying control

- 10 jobs on an IBM 20-qubit chip on 3/23/19
- Each job consists of 128 identical experiments:

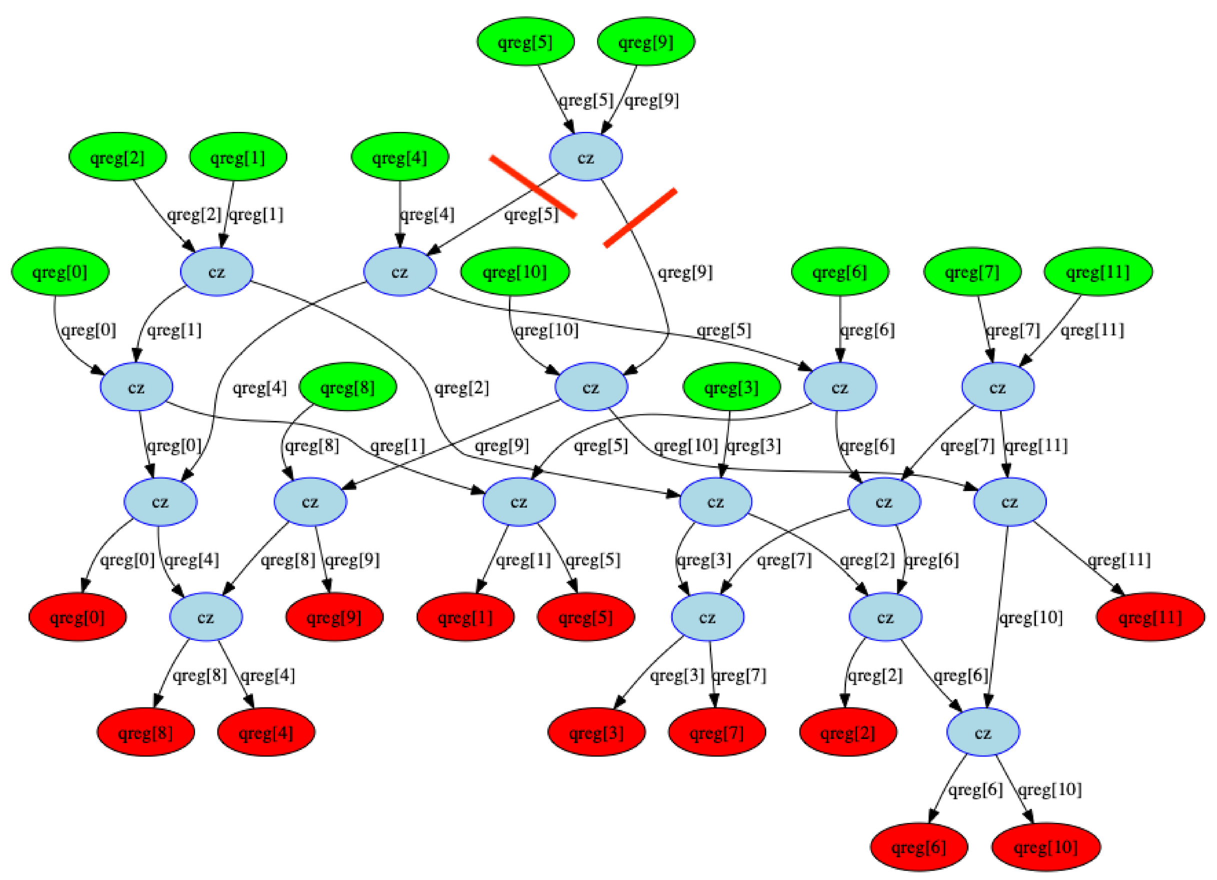
  repeat 8192 **times**
  Prepare zero state on qubit 0
  Read out

- Counts drift significantly over time:



Currently developing a Bayesian method for optimizing functions with noise that depends on time and input parameters

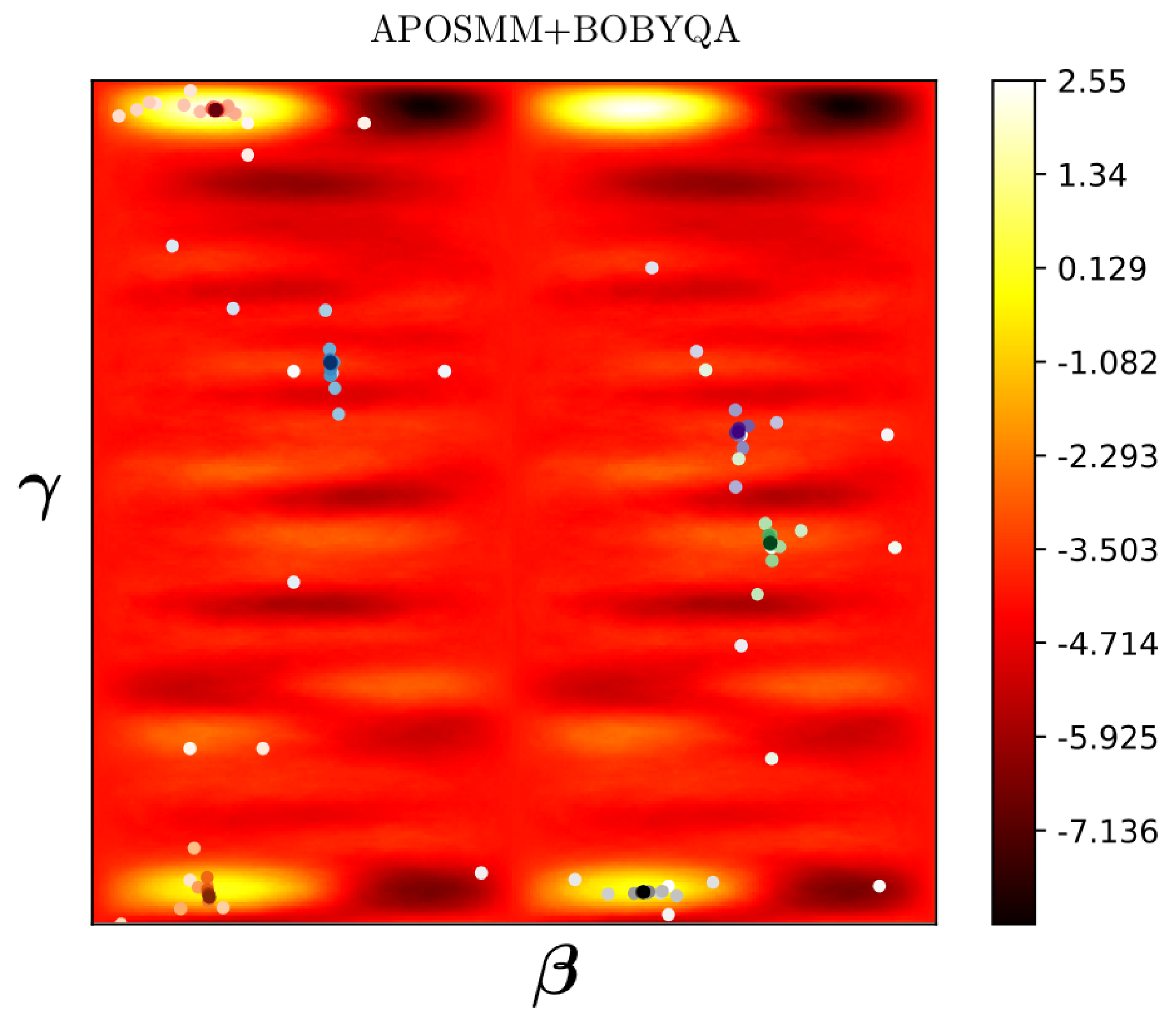
Circuit cutting



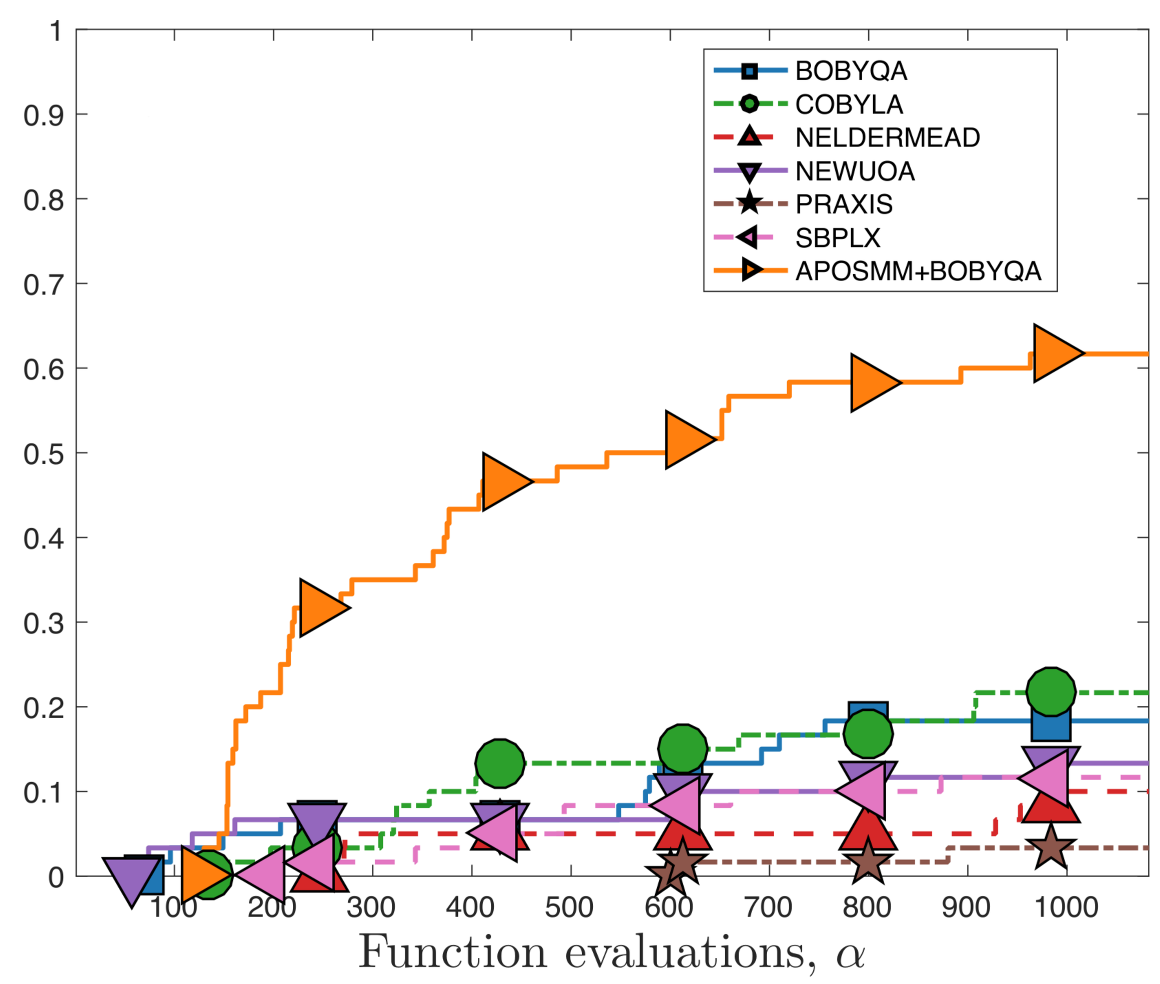
May want to cut a circuit because:

- The number of physical qubits on NISQ devices limit the size of executable quantum circuits
- Hardware noise and qubit decoherence degrade longer output from longer circuits
- Need to identify nontrivial connectivity in a quantum circuit
- Want to apply intermediate error detection
- Wish to classically simulate a portion of a circuit

Comparing classical optimizers on QAOA



QAOA objective surface and points sampled by APOSMM+BOBYQA for a graph clustering problem



Data profiles for different classical optimizers on benchmark graph clustering problems with $p = 2$

Clustering problem

$$\underset{v_{i,c}}{\text{minimize}} \sum_{c=1}^{r} 2^{d_c + e_c \log_2 6}$$
$$\text{subject to} \quad d_c \le D$$
$$\text{and other constraints}$$

where

- d_c is the number of qubits required for cluster c
- e_c is the number of incoming edges to cluster c
- $x_{i,c} = 1$ if vertex i is assigned to cluster c

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